MAT441 Homework Assignment 2

1. Show that \(|p| - |q| \leq |p - q|\) for \(p, q \in \mathbb{R}^n\).

2. Use the law of cosines in the plane and the properties of the norm and scalar product to verify that
\[ p \cdot q = |p||q| \cos \theta \]
for \(p, q \in \mathbb{R}^n\).

3. Find the equation of the hyperplane in 4-space which goes through the point \(p_0 = (0, 1, -2, 3)\) and is perpendicular to the vector \(a = (4, 3, 1, -2)\).

4. If the angle between two hyperplanes is defined as the angle between their normals, are the hyperplanes \(3x + 2y + 4z - 2w = 5\) and \(2x - 4y + z + w = 6\) orthogonal?

5. Write the parametric equations of the line through \((2, 3, -1, 1)\) which is perpendicular to the hyperplane \(3x + 2y - 4z + w = 0\).

6. Let \(\ell\) be the line determined by the two points \(p\) and \(q\). Let \(P = \lambda p + (1 - \lambda)q\). Show that, when \(\lambda > 1\),
\[ |P - p| + |p - q| = |P - q|, \]
and interpret this geometrically.

7. Show that the intersection of two convex sets is convex but that the union of convex sets does not have to be convex.