Two-Way ANOVA

PSY 360
Introduction to Statistics for the Behavioral Sciences

Two-Way ANOVA: Introduction

- The two-way ANOVA uses two factors, variables that combine to form the groups. The factors may or may not be independent variables.
- The groups formed by combining levels/values of the factors are called cells, and the means of the observations in these cells are called cell means.
- We have three F-tests in a two-way ANOVA, one for each of the two factors by themselves, and one for the interaction of the two factors.

Two-Way ANOVA: Factors vs. Levels

- Each two-way ANOVA always has two factors, but each of these factors can have different numbers of levels (levels are the values of the factors). Here are several different two-way layouts:
Two-Way ANOVA: Logic

- The logic of the two-way ANOVA is the same as that for the one-way: for each of the three F-tests, form an F-ratio of two sample variances. For each F, if $H_0$ is true, both variances should be equal and the average F will be about 1.
- For each F, if $H_0$ is false,
  - We expect numerator>denominator,
  - We expect average F>1.
- And we reject $H_0$ if $F>F_{crit}$.
- The difference is that the two-way ANOVA is more complex: there are three Fs.
  - The effects of the factors are called main effects (and the F for the interaction).

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Two-Way ANOVA: F-tests

- **Situation/hypothesis**
  - Two factors, J levels of A, K levels of B, n observations per cell
  - For A, $H_{0j} = \mu_1 = \mu_2 = \ldots = \mu_J$
  - For B, $H_{0k} = \mu_1 = \mu_2 = \ldots = \mu_K$
  - For AB, $H_{0jk}$ no interaction effect

- **Test statistic**
  - $F_A = MS_A/MS_W$
  - $F_B = MS_B/MS_W$
  - $F_{AB} = MS_{AB}/MS_W$

- **Distribution**
  - Similar to One-Way ANOVA
  - $F_{J-1,JK(n-1)}$

- **Assumptions**
  - 1. Populations are normal
  - 2. Equal population variances for each cell
  - 3. Observations are independent

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Two-Way ANOVA: Introduction

- Example: runners and cyclists randomly assigned to one of three amounts of time to hold a hamstring stretch, tested for flexibility after six weeks. Sport and time are the factors, we call this a 2X3 ANOVA, and there are 6 cells.
- So we will have an F-ratio for sport, an F-ratio for time, and an F-ratio for the interaction of sport and time.

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Two-Way ANOVA: Logic

- Notation: \( n=\# \text{ obs. per cell}, J=\# \text{ levels of A}, K=\# \text{ levels of B}, N=nJK \)
- Each of the three F's is formed as a ratio of two sample variances: the numerator will be the MS for the effect tested (MS_A, MS_B, or MS_AB), the denominator will be MS_W
- Hypotheses:
  - For A (e.g. Sport)
    - \( H_0: \mu_1 = \mu_2 = \ldots = \mu_J \)
    - \( H_1: \text{any differences in } \mu_j \text{'s} \)
  - For B (e.g. Time)
    - \( H_0: \mu_1 = \mu_2 = \ldots = \mu_K \)
    - \( H_1: \text{any differences in } \mu_k \text{'s} \)
  - For interaction (not easily expressed in terms of } \mu \text{'s),
    - \( H_0: \text{no interaction effect} \)
    - \( H_1: \text{some interaction effect} \)

Interpreting the Two-Way ANOVA Plot

1. If the lines are not parallel, then interaction is indicated (may or may not be significant, depending on chance variability)
2. If the midpoints of the lines are not equal, then an A effect is indicated
3. If the visual average (middle) of the points (cell means) above each level of Factor B are not equal, then a B effect is indicated

Two-Way ANOVA: Interaction

- Symptoms of arthritis frequently include stiffness and joint pain. A new drug helps only women who experience stiffness, not women with joint pain nor men with either symptom.
- Using a rating of the drug that increases as effectiveness of the drug increases, these results would look like this:

- Gender (M or F) is interacting with symptom (Stiffness or Joint Pain).
Two-Way ANOVA: Interaction

Plots of cell means showing the three F-tests (assumes that $MS_w$ is small so any observed difference is significant).

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Two-Way ANOVA: Test Statistics

The test statistics are F-ratios, $F_A=MS_A/MS_w=(SS_A/df_A)/(SS_W/df_W)$, where $df_A=J-1$ and $df_W=JK(n-1)$

$F_B=MS_B/MS_w=(SS_B/df_B)/(SS_W/df_W)$, where $df_B=K-1$ and $df_W=JK(n-1)$

$F_{AB}=MS_{AB}/MS_w=(SS_{AB}/df_{AB})/(SS_W/df_W)$, where $df_{AB}=(J-1)(K-1)$ and $df_W=JK(n-1)$

If $n=20$, $J=3$, and $K=4$, compute the df:

- $df_A=J-1=3-1=2$
- $df_B=K-1=4-1=3$
- $df_{AB}=(J-1)(K-1)=(3-1)(4-1)=2(3)=6$
- $df_w=JK(n-1)=(3)(4)(20-1)=12(19)=228$
Two-Way ANOVA: F Distributions

- Each F-statistic in a two-way ANOVA has its own, possibly different, F distribution and F critical value.
  - $F_A$ is distributed as $F_{J-1, JK(n-1)}$
  - $F_B$ is distributed as $F_{K-1, JK(n-1)}$
  - $F_{AB}$ is distributed as $F_{(J-1)(K-1), JK(n-1)}$